Probability & Bridge

NKy Summer Getaway Sectional
August 12, 2017
Goals

• Practical bridge advice
• Improve how we think at the table
• Get better results in tough contracts

• **NOT**: combinatorial mathematics or statistical equations.

\[
\binom{N}{n} = \frac{N!}{n!(N-n)!}
\]
Simple Chances

- Flip a coin
- Roll a die
- Take a finesse
Flip a Coin

- Coin has two sides (2 Total cases)
- One side is up (1 Specific Case): Heads or Tails
- *a priori* probability = \( \frac{1}{2} = 50\% \)

- Each coin toss is INDEPENDENT of the prior event (Coins have no memories)

- **Probability of success for 2 independent events is the product of the probability of each:**
  - Two coins giving heads (HH): \( \frac{1}{2} \times \frac{1}{2} = 25\% \)
  - 3 Coins giving Heads (HHH): 25\% \times \frac{1}{2} = 12.5\% \) etc...
Coin Quiz

• Which sequence of 10 coin tosses is more likely?

  Sequence A: HHHHHHHHHHH  0.0977%
  Sequence B: THTTHTTHHHT  0.0977%
Rolling a Die

• A standard die has 6 sides – 6 Total cases
• One side shows up – 1 Specific Case.
• The roll of any one die each number has an equal probability of $1/6 = 16.67\%$
• Each role is INDEPENDENT (die has no memory)

• **Q:** With two fair dice, what is the probability of rolling a 7?
Rolling a 7

• Outcome table (6x6=36 Total Cases)

• Frequency Table:

<table>
<thead>
<tr>
<th>#</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cases</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>%</td>
<td>3.84</td>
<td>7.69</td>
<td>11.54</td>
<td>15.38</td>
<td>19.23</td>
<td>26.09</td>
<td>19.23</td>
<td>15.38</td>
<td>11.54</td>
<td>7.69</td>
<td>3.84</td>
<td>100</td>
</tr>
</tbody>
</table>

Rolling a 7 is 26.09%

Craps (2 or 12) is 7.69%, the SUM of 2% and 12% (3.84 + 3.84).

For independent events, A and B is the **product** $P_A \times P_B$, while A or B is the **sum** $P_A + P_B$. 
When is a finesse like a coin flip?

- When we lack INFORMATION!!!
- 2 Cases: Win or lose
- Just like coin: Heads or Tails
- Therefore Finesse is 50%, lacking other information
Bridge Hands – BIG NUMBERS

• 635,013,559,600 - # of ways to deal 13 cards.
• 53,644,737,765,488,792,839,237,440,000 - the number of possible ways to deal all 52 cards, 13 at a time.
• Odds of 4 players being dealt all 13 cards in one suit:
  1 in 2,235,197,406,895,366,368,301,559,999
Which Hand is More Likely?

♠ AKQJ1098765432
♠ AK32
♥ K984
♦ Q10
♣ J107
<table>
<thead>
<tr>
<th>What I gave you:</th>
<th>What you saw:</th>
<th>What you assumed:</th>
</tr>
</thead>
<tbody>
<tr>
<td>♠ AK32</td>
<td>♠ AKxx</td>
<td>♠ □□□□□□□□□□</td>
</tr>
<tr>
<td>♥ K984</td>
<td>♥ Kxxx</td>
<td>♥ □□□□□□□□□□</td>
</tr>
<tr>
<td>♦ Q10</td>
<td>♦ Q10</td>
<td>♦ □□□□□□□□□□</td>
</tr>
<tr>
<td>♣ J107</td>
<td>♣ J10x</td>
<td>♣ □□□□□□□□□□</td>
</tr>
</tbody>
</table>
SUIT SPLITS

- # of specific cases / # Total Cases (approximately)
- # Total Cases = $2^m$ (where $m$=# missing cards)
- Study 2 – 7 missing cards (4 – 128 Tot. Cases)

<table>
<thead>
<tr>
<th>Split</th>
<th>0</th>
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<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Split Missing:</td>
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<tr>
<td>7</td>
<td>0.5</td>
<td>7</td>
<td>30.5</td>
<td>62</td>
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<tr>
<td>6</td>
<td>1.5</td>
<td>14.5</td>
<td>48</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>28</td>
<td>68</td>
<td></td>
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<td>4</td>
<td>10</td>
<td>50</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>78</td>
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<td></td>
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<tr>
<td>2</td>
<td>48</td>
<td>52</td>
<td></td>
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</table>
## DROP Missing Honors

<table>
<thead>
<tr>
<th>Honor Drop Missing</th>
<th>%</th>
<th>H</th>
<th>Hx</th>
<th>Hxx</th>
<th>TOT</th>
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<tr>
<td></td>
<td>8</td>
<td>0.4</td>
<td>4</td>
<td>18</td>
<td>22.4</td>
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<td></td>
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<td>1</td>
<td>9</td>
<td>27</td>
<td>37</td>
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<tr>
<td></td>
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<td>16</td>
<td>36</td>
<td>54.4</td>
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<tr>
<td></td>
<td>2</td>
<td>52</td>
<td>48</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Suit Combinations

• How to play suits – **wrong** when done alone – **right** when done in the **context of whole hand**.
• **Know # tricks needed.**
• **Vacant Spaces** 13 each, reduced by **information**.
Suit Combinations

1. **K3** opposite **6710QA** Do you finesse for the 10? Why/Not?

2. **AJ975 -- 6810K** - You play the K. LHO plays the 2 RHO the 3. You play the 6 to dummy. LHO plays the 4. **Finesse or drop?**

3. **AQ97** opposite **810K** - You play the 10 to the Q and the 7 to the K, RHO playing 2,4. LHO playing 3, 5. Now you continue **the 8** and LHO plays the 6. **Finesse or drop?**

4. **AJ1074** opposite **52** (need 3 tricks)

5. **AKQ74** opposite **52** (need 4 tricks; Need 5 tricks) **NO SIDE ENTRIES.**
Suit Combination 1

- **K3** opposite **6710QA** Do you finesse for the 10? Why/Not?
  - Absent information the finesse is worth 50%.
  - If we are looking for the J, then we can win when the J is singleton, doubleton or Jxx in either hand.
  - Combining those chances results in $2.4 + 16 + 36 = 54.4\%$ so cashing tops is better.

What Information would make you change your play?
- Count of the hand $\rightarrow$ split known
- # Tricks needed from this suit
- Avoid having RHO on lead
- Can ruff out the suit
Suit Combination 2

- **AJ975 -- 6810K** - You play the K. LHO plays the 2. RHO plays the 3. You play the 6 to dummy. LHO plays the 4. **Finesse or drop?**
  - Any 2-2 break is 40% while any 3-1 break is 50%.
  - 2-2 has 12 cases. 3-1 has only 8. So the specific case for 3-1 is less likely (absent additional information).
  - The Qxx w/ LHO is 6.21%. The Qx with RHO is 6.78%.
  - The ratio 6.78/13 = 52.2%.
  - Vacant spaces says LHO has 11 while RHO has 12 before declarer’s choice. 12/23 = 52.2% the Q is with RHO.

What Information would make you change your play?
Suit Combination 3

• **AQ97 opposite 810K** - You play the 10 to the Q and the 7 to the K, RHO playing 2,4. LHO playing 3, 5. Now you continue the 8 and LHO plays the 6. *Finesse or drop?*

  – You have seen 3 insignificant cards from LHO and 2 from RHO.
  – That leaves 10 spaces for LHO and 11 for RHO.
  – Therefore the probability that the J is with LHO is $11/(10+11) = 52.4\%$. **DROP**

What Information would make you change your play?
Suit Combination 4

- **AJ1074** opposite **52** – **Goal: 3 tricks**
  - We are missing the **KQ9863**
  - From the chart, 3-3 happens 36% of the time and 4-2 happens 48%.
  - Missing 6 cards there are \(2^6 = 64\) total cases.
  - 6 cards taken 3 at a time counts to 20
  - 6 cards take 2 (or four) at a time counts to 30

- Any 3-3 means we win 3 tricks. Any 1-5 or 0-6 and we fail. Ignore these.
- 4-2/2-4 is where we can gain advantage. A finesse, and playing A then small are equivalent for all Hxxx-Hx/Hx-Hxxx. The finesse gains for all HHxx-xx, 6 cases more than A then x. But loses for the case xxxx-HH, net 5 cases different. **Finesse!**

<table>
<thead>
<tr>
<th>3 Tricks</th>
<th>Totals</th>
<th>64</th>
<th>16100</th>
<th>100.00</th>
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<tr>
<td><strong>A</strong> 2 finesses</td>
<td></td>
<td>43</td>
<td>11700</td>
<td>72.67</td>
</tr>
<tr>
<td><strong>B</strong> A and then small</td>
<td></td>
<td>38</td>
<td>10400</td>
<td>64.60</td>
</tr>
</tbody>
</table>

- Notice if we hold AJ10542 opposite 7, we are missing the same 6 cards but can take only **one** finesse. Now we are better playing A then x instead of finessing for the 16 cases for Hxxx-Hx/Hx-Hxxx.
Suit Combination 5

- **AKQ74** opposite 52 (need 4 tricks; Need 5 tricks)
- Needing 5 tricks, we play top down, for a 36% chance (3-3 split).
- Needing 4 tricks we can do better. If we duck the first trick we will get 4 tricks if the suit splits 3-3 (36%) or 4-2/2-4 (48%). This improves our chances to 84%. Much better than playing the suit top down (remember we have no outside entry).
Suit Split Probability

Richard Pavlicek
Bridge Site: http://www.rpbridge.net/

Case: Missing 6 cards including the Q

Best way to answer the question “WHY??”

Now the fun starts!

<table>
<thead>
<tr>
<th>#</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>West</th>
<th>East</th>
<th>Ways</th>
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<th>Percent</th>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Qxxxxxx</td>
<td>—</td>
<td>1</td>
<td>24</td>
<td>0.75</td>
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<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Qxxxxx</td>
<td>x</td>
<td>5</td>
<td>195</td>
<td>6.06</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Qxxx</td>
<td>xx</td>
<td>10</td>
<td>520</td>
<td>16.15</td>
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<tr>
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<td></td>
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<td></td>
<td></td>
<td>Qxx</td>
<td>xxx</td>
<td>10</td>
<td>572</td>
<td>17.76</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Qx</td>
<td>xxx</td>
<td>5</td>
<td>260</td>
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</tr>
<tr>
<td>6</td>
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<td></td>
<td></td>
<td>Q</td>
<td>xxx</td>
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<td>39</td>
<td>1.21</td>
</tr>
<tr>
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<td></td>
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<td></td>
<td></td>
<td>xxxxx</td>
<td>Q</td>
<td>1</td>
<td>39</td>
<td>1.21</td>
</tr>
<tr>
<td>8</td>
<td></td>
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<td>xxxx</td>
<td>Qx</td>
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<td>xx</td>
<td>Qxxx</td>
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<td>520</td>
<td>16.15</td>
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<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
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<td>x</td>
<td>Qxxxx</td>
<td>5</td>
<td>195</td>
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<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
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<td>—</td>
<td>Qxxxxx</td>
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Goal to win

<table>
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<th>100.00</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>Q, Qx, Qxx onside</td>
<td>3</td>
<td>16</td>
<td>871</td>
</tr>
<tr>
<td>B</td>
<td>Ruff the 4th Club</td>
<td>3</td>
<td>16</td>
<td>739</td>
</tr>
<tr>
<td>C</td>
<td>Play off 3 rounds of Clubs</td>
<td>6</td>
<td>32</td>
<td>1742</td>
</tr>
<tr>
<td>D</td>
<td>Q drops in 2 rounds</td>
<td>4</td>
<td>12</td>
<td>598</td>
</tr>
</tbody>
</table>
Combining Chances

• ♠ AJ1097 opposite 543

What is the probability you can score 4 tricks?
  – p Both ♠ A&K are onside: 24%
  – p Honors are split 52%
  – p Both honors are on your right 24%

P Success = 24% + 52% = 76%

• We Need: Finesse in Suit A, and if that fails a 3-3 break in suit B.
What is the probability we make our contract?
  – 50% Finesse wins + 50% Finesse loses X (36% 3-3 split) = 68%
Analyze 1\textsuperscript{st} – Plan 2\textsuperscript{nd}

### The Whole Bridge Hand
- Use ALL your information – Bidding & Play
- **Start** with a flexible picture of declarer/opponent
- Count hand **winners and losers** (“off the top”) and **SLOW LOSERS**
- Count **entries**
- Count **stoppers** in threat suits.
- Count **HCP** - Your total and their total
- **Combine your chances** – Source of Tricks/Trick Packets
- Avoid the **DANGER HAND**. Assume perfect defense.
- **Modify plan** as you learn - Show-outs are **GOLD**

### Use All Information

#### Common Inferences
- Opening bids show 12 HCP + and 5+ Cards in a Major.
- 1 NT is typically 15-17.
- Weak 2 for 7-8 HCP and 6 cards
- a 3-bid less (~6) and 6-7 cards

An **INference** is what we judge

**information** is what we see and know. (Show outs are **information**)
Tips

Combining Chances (Mutually Exclusive events)
• Plan for failure – Stay ALIVE.
• Find chances that create options
• Cash winners in your long side suit (drop honors)
• Finesse long suits into safe hands when necessary.
• Avoid finesses completely if possible
• Leave short suits (no extra chances) until the end.

Steve’s Tips:
• Always choose the plan with the best probability
• Find a good plan? LOOK AGAIN. FIND A BETTER ONE
• When faced with equal choices, choose the option that allows you to STAY ALIVE longest (Take more chances)
• Any Chance is better than NO Chance
• NEVER take a PRACTICE FINESSE.
A Simple Hand?

Contract: 4♠, Opponents pass throughout

♠ QJ432  N  ♠ 1098
♥ AK2    W  ♥ 1064
♦ Q3     E  ♦ AK65
♣ J456   S  ♣ KQ2

Lead ♥3

Analysis:
Winners: 5  Losers: 3 Fast, 1 Slow
Entries: W3 and E2
Stoppers: ♥2,
Source of Tricks: ♠(3), ♣(2)
Improving Your Plan

Contract 6♥, no opposing bidding

♠ AQ     ♠ 98
♥ AKJ72   ♥ Q10654
♦ AQ
♣ J456   ♣ AK10

Lead ♥3

ANAYLSIS:
Winners: 9  Losers: 0 Fast, 3 Slow
Entries:  W5 and E3
Stoppers: 1♠ 1♦ 2♣
Source of Tricks: ♠ (3); ♦(2)
Plan 6♥
A Novice (or *finesse-aholic*)

- sees 3 finesses, draw trumps in 2-4 rounds and begin.
- 3 Finesses here are independent (different suits, different players) so the odds of all 3 are \( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \) or 12.5%.
- They need only 2 of the 3 finesses. How do you calculate the probability? Think: 2 winning finesses is the same case as one losing finesse or *50%*. 
Improving your plan 2

Intermediate Player:

• After pulling trumps if the ♠ finesse works and they split 3-3, they can pitch a losing ♠ on the long ♠,

Combining chances that way means:

– 50% ♠ finesse x 36% 3-3- split = 18%
– 50% ♦ Finesse x 82% remaining = 41% or
– TOTAL CHANCE: 59%

a useful improvement.
Improving your plan 3

Expert Player:

- Cash the ♠ AK = 18% +
- % Time North has 0, 1, 2 ♠ = 26% (NOT Q, Qx)
- ♦ Finesse (50% x 56%) = 28%
- TOTAL 72+%  

• **Look deeper!** Treat the hand as “one of 2 finesses”: Finesse ♠. If win, cash ♠ tops. If no ♠Q, finesse ♦. If lose, long ♠ goes away on 3rd ♦. Likewise if ♠ finesse loses, we need only the ♦ finesse to win (Pitch the losing ♠ on the ♣) - a 75% play.
Bonus Problem

6♠, No opposition bids. What is the likelihood of success?
Trumps split 2-1

♠ QJ432
♥ AK
♦ KJ3
♣ AJ4

♠ AK987
♥ 64
♦ A65
♣ K102

Lead ♥3

Which finesse do you take first? Second? Why?

**ANSWER:** Take NO Finesses. Draw 2 rounds of Trump and 2 rounds of ♥. Then play off ♦AKJ in that order. No matter who wins they have to either give you a free ♠ finesse or a ruff sluff – either way we lose only 1 ♦ trick.

**PROBABILITY OF SUCCESS: 100%**
ANSWER: Take NO Fineses. Draw 3 rounds of Trump and 2 rounds of ♥. Then play off ♦AKJ in that order. No matter who wins they have to either give you a free ♣ finesse or a ruff sluff – either way we lose only 1 ♦ trick.

**PROBABILITY OF SUCCESS: 100%**
Other Uses for “p”

- When to bid Game, Small Slam, and Grand Slam
  - The likelihood of success must match or exceed breakeven
  - Breakeven – what you win equals what you lose.
# Game, Slam & Grand Odds

<table>
<thead>
<tr>
<th>Type</th>
<th>Games</th>
<th>Small Slam</th>
<th>Grand Slam</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Type</td>
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<td>NV</td>
</tr>
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<td></td>
<td>NV</td>
<td>45%</td>
<td>50%</td>
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</tbody>
</table>

*If opponents bid game, then bidding a Grand Slam is Poor.* With 12 tricks, a slam gains +11NV & +13V IMPS, the grand loses -11NV Imps and -13V, swinging -22NV & -26V Imps. Avoid grand slams when they only bid game. Need 14 tricks.
<table>
<thead>
<tr>
<th>Useful %:</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chances you’ll have fun playing Bridge</td>
<td>100</td>
</tr>
<tr>
<td>Need 1 of 2 finesses</td>
<td>75</td>
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<tr>
<td>Missing cards split 3-2</td>
<td>68</td>
</tr>
<tr>
<td>Missing cards split 4-3</td>
<td>62</td>
</tr>
<tr>
<td>Q drops in 3 rounds when holding 7 cards</td>
<td>54.4</td>
</tr>
<tr>
<td>Pure finesse</td>
<td>50</td>
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<tr>
<td>Need 2 finesses of 3 available</td>
<td>50</td>
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<tr>
<td>Q drops in 3 rounds when holding 6 cards</td>
<td>37</td>
</tr>
<tr>
<td>Suit splits 3-3</td>
<td>36</td>
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<tr>
<td>Need 3 finesses of 4 available.</td>
<td>31</td>
</tr>
<tr>
<td>Need 2 finesse of 2 available</td>
<td>25</td>
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<tr>
<td>Need finesse &amp; 3-3 split</td>
<td>18</td>
</tr>
<tr>
<td>Need 3 finesses</td>
<td>12.5</td>
</tr>
</tbody>
</table>
References

2. Richard Pavlicek Bridge Site: http://www.rpbridge.net/
THANK YOU FOR YOUR KIND ATTENTION!

Steve Moese
See you at the tables!