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Wikipedia: http://en.wikipedia.org/wiki/Bridge_probabilities

#cards	Distribution	Probability	Combinations	Individual Prob
2	1 - 1	0.52	2	0.26
	2 - 0	0.48	2	0.24
3	2 - 1	0.78	6	0.13
	3 - 0	0.22	2	0.11
4	2 - 2	0.41	6	0.0678~
	3 - 1	0.50	8	0.0622~
	4 - 0	0.10	2	0.0478~
5	3 - 2	0.68	20	0.0339~
	4 - 1	0.28	10	0.02826~
	5 - 0	0.04	2	0.01956~
6	3 - 3	0.36	20	0.01776~
	4 - 2	0.48	30	0.01615~
	5 - 1	0.15	12	0.01211~
	6 - 0	0.01	2	0.00745~
7	4 - 3	0.62	70	0.00888~
	5 - 2	0.31	42	0.00727~
	6 - 1	0.07	14	0.00484~
	7 - 0	0.01	2	0.00261~
8	4 - 4	0.33	70	0.00467~
	5 - 3	0.47	112	0.00421~
	6 - 2	0.17	56	0.00306~
	7 - 1	0.03	16	0.00178~
	8 - 0	0.00	2	0.00082~

Probability of suit distributions in two hidden hands

This table represents the different ways that two to thirteen particular cards may be distributed, or may *lie* or *split*, between two unknown 13-card hands (before the bidding and play, or *a priori*). The table also shows the number of combinations of particular cards that match any numerical split and the probabilities for each combination. These probabilities follow directly from the law of Vacant Places.

Probability of HCP distribution

High Card Points (HCP) are usually counted using the Milton Work scale of 4/3/2/1 points for each Ace/King/Queen/Jack respectively. The a priori probabilities that a given hand contains no more than a specified number of HCP is given in the table below[1]. To find the likelihood of a certain point range, one simply subtracts the two relevant cumulative probabilities. So, the likelihood of being dealt a 12-19 HCP hand (ranges inclusive) is the probability of having at most 19 HCP minus the probability of having at most 11 HCP, or: 0.986 − 0.652 = 0.334.[2]

HCP	Prob	HCP	Prob	HCP	Prob	HCP	Prob	HCP	Prob
0	0.0036	8	0.3748	16	0.9355	24	0.9995	32	1.0000
1	0.0115	9	0.4683	17	0.9591	25	0.9998	33	1.0000
2	0.0251	10	0.5624	18	0.9752	26	0.9999	34	1.0000
3	0.0497	11	0.6518	19	0.9855	27	1.0000	35	1.0000
4	0.0882	12	0.7321	20	0.9920	28	1.0000	36	1.0000
5	0.1400	13	0.8012	21	0.9958	29	1.0000	37	1.0000
6	0.2056	14	0.8582	22	0.9979	30	1.0000		
7	0.2858	15	0.9024	23	0.9990	31	1.0000		

Hand pattern probabilities

A *hand pattern* denotes the distribution of the thirteen cards in a hand over the four suits. In total 39 hand patterns are possible, but only 13 of them have an *a priori* probability exceeding 1%. The most likely pattern is the 4-4-3-2 pattern consisting of two four-card suits, a three-card suit and a doubleton.

Note that the hand pattern leaves unspecified which particular suits contain the indicated lengths. For a 4-4-3-2 pattern, one needs to specify which suit contains the three-card and which suit contains the doubleton in order to identify the length in each of the four suits. There are four possibilities to first identify the three-card suit and three possibilities to next identify the doubleton. Hence, the number of *suit permutations* of the 4-4-3-2 pattern is twelve. Or, stated differently, in total there are twelve ways a 4-4-3-2 pattern can be mapped onto the four suits.

Below table lists all 39 possible hand patterns, their probability of occurrence, as well as the number of suit permuatation for each pattern. The list is ordered according to likelihood of occurrence of the hand patterns.[3]

Pattern	Probability	#	Pattern	Probability	#	Pattern	Probability	#
4-4-3-2	0.2155	12	5-5-3-0	0.0090	12	9-2-1-1	0.00018	12
5-3-3-2	0.1552	12	6-5-1-1	0.0071	12	9-3-1-0	0.00010	24
5-4-3-1	0.1293	24	6-5-2-0	0.0065	24	9-2-2-0	0.000082	12
5-4-2-2	0.1058	12	7-2-2-2	0.0051	4	7-6-0-0	0.000056	12
4-3-3-3	0.1054	4	7-4-1-1	0.0039	12	8-5-0-0	0.000031	12
6-3-2-2	0.0564	12	7-4-2-0	0.0036	24	10-2-1-0	0.000011	24
6-4-2-1	0.0470	24	7-3-3-0	0.0027	12	9-4-0-0	0.000010	12
6-3-3-1	0.0345	12	8-2-2-1	0.0019	12	10-1-1-1	0.000004	4
5-5-2-1	0.0317	12	8-3-1-1	0.0012	12	10-3-0-0	0.0000015	12
4-4-4-1	0.0299	4	7-5-1-0	0.0011	24	11-1-1-0	0.0000002	12
7-3-2-1	0.0188	24	8-3-2-0	0.0011	24	11-2-0-0	0.0000001	12
6-4-3-0	0.0133	24	6-6-1-0	0.00072	12	12-1-0-0	0.000000003	12
5-4-4-0	0.0124	12	8-4-1-0	0.00045	24	13-0-0-0	0.000000000006	4

The 39 hand patterns can by classified into four *hand types*: balanced hands, three-suiters, two suiters and single suiters. Below table gives the *a priori* likelihoods of being dealt a certain hand-type.

Hand type	Patterns	Probability
Balanced	4-3-3-3, 4-4-3-2, 5-3-3-2	0.4761
2-suiter	5-4-2-2, 5-4-3-1, 5-5-2-1, 5-5-3-0, 6-5-1-1, 6-5-2-0, 6-6-1-0, 7-6-0-0	0.2902
1-suiter	6-3-2-2, 6-3-3-1, 6-4-2-1, 6-4-3-0, 7-2-2-2, 7-3-2-1, 7-3-3-0, 7-4-1-1, 7-4-2-0, 7-5-1-0, 8-2-2-1, 8-3-1-1, 8-3-2-0, 8-4-1-0, 8-5-0-0, 9-2-1-1, 9-2-2-0, 9-3-1-0, 9-4-0-0, 10-1-1-1, 10-2-1-0, 10-3-0-0, 11-1-1-0, 11-2-0-0, 12-1-0-0, 13-0-0-0	0.1915
3-suiter	4-4-4-1, 5-4-4-0	0.0423

Alternative grouping of the 39 hand patterns can be made either by longest suit or by shortest suit. Below tables gives the *a priori* chance of being dealt a hand with a longest or a shortest suit of given length.

Longest suit	Patterns	Probability
4 card	4-3-3-3, 4-4-3-2, 4-4-4-1	0.3508
5 card	5-3-3-2, 5-4-2-2, 5-4-3-1, 5-5-2-1, 5-4-4-0, 5-5-3-0	0.4434
6 card	6-3-2-2, 6-3-3-1, 6-4-2-1, 6-4-3-0, 6-5-1-1, 6-5-2-0, 6-6-1-0	0.1655
7 card	7-2-2-2, 7-3-2-1, 7-3-3-0, 7-4-1-1, 7-4-2-0, 7-5-1-0, 7-6-0-0	0.0353
8 card	8-2-2-1, 8-3-1-1, 8-3-2-0, 8-4-1-0, 8-5-0-0	0.0047
9 card	9-2-1-1, 9-2-2-0, 9-3-1-0, 9-4-0-0	0.00037
10 card	10-1-1-1, 10-2-1-0, 10-3-0-0	0.000017
11 card	11-1-1-0, 11-2-0-0	0.0000003
12 card	12-1-0-0	0.000000003
13 card	13-0-0-0	0.000000000006

Shortest suit	Patterns	Probability
2 card	4-3-3-3	0.1054
2-ton	4-4-3-2, 5-3-3-2, 5-4-2-2, 6-3-2-2, 7-2-2-2	0.5380
1-ton	4-4-4-1, 5-4-3-1, 5-5-2-1, 6-3-3-1, 6-4-2-1, 6-5-1-1, 7-3-2-1, 7-4-1-1, 8-2-2-1, 8-3-1-1, 9-2-1-1, 10-1-1-1	0.3055
Void	5-4-4-0, 5-5-3-0, 6-4-3-0, 6-5-2-0, 6-6-1-0, 7-3-3-0, 7-4-2-0, 7-5-1-0, 7-6-0-0, 8-3-2-0, 8-4-1-0, 8-5-0-0, 9-2-2-0, 9-3-1-0, 9-4-0-0, 10-2-1-0, 10-3-0-0, 11-1-1-0, 11-2-0-0, 12-1-0-0, 13-0-0-0	0.0512

Number of possible deals

In total there are 53,644,737,765,488,792,839,237,440,000 (5.36 x 10^28) different deals possible, which is equal to $52! / (13!)^4$. The immenseness of this number can be understood by answering the question "*How large an area would you need to spread all possible bridge deals if each deal would occupy only one square millimeter?*". The answer is: *an area more than a hundred million times the total area of the earth*.

Obviously, the deals that are identical except for swapping—say—the ♥2 and the ♥3 would be unlikely to give a different result. To make the irrelevance of small cards explicit (which is not always the case though), in bridge such small cards are generally denoted by an 'x'. Thus, the "number of possible deals" in this sense depends of how many non-honour cards (2, 3, .. 9) are considered 'indistinguishable'. For example, if 'x' notation is applied to all cards smaller than ten, then the suit distributions A987-K106-Q54-J32 and A432-K105-Q76-J98 would be considered identical. The table below [4] gives the number of deals when various numbers of small cards are considered indistinguishable.

Suit composition	Number of deals
AKQJT9876543x	53,644,737,765,488,792,839,237,440,000
AKQJT987654xx	7.811,544,503,918,790,990,995,915,520
AKQJT98765xxx	445,905,120,201,773,774,566,940,160
AKQJT9876xxxx	14,369,217,850,047,151,709,620,800
AKQJT987xxxxx	314,174,475,847,313,213,527,680
AKQJT98xxxxxx	5,197,480,921,767,366,548,160
AKQJT9xxxxxxx	69,848,690,581,204,198,656
AKQJTxxxxxxxx	800,827,437,699,287,808
AKQJxxxxxxxxx	8,110,864,720,503,360
AKQxxxxxxxxxx	74,424,657,938,928
AKxxxxxxxxxxx	630,343,600,320
Axxxxxxxxxxxx	4,997,094,488
xxxxxxxxxxxxx	37,478,624

Note that the last entry in the table (37,478,624) corresponds to the number of different distributions of the deck (the number of deals when cards are only distinguished by their suit).

References

1. ^ *a* *b* "Mathematical Tables" (Table 4). In Henry G. Francis, et al., eds., *The Official Encyclopedia of Bridge*, 5th edition. (ISBN 0943855-48-9)
2. ^ Richard Pavlicek. "High Card Expectancy." link
3. ^ Richard Pavlicek. "Against All Odds." link
4. ^ Counting Bridge Deals, Jeroen Warmerdam